1. Relations and Functions

Exercise 1.1

Concept corner

Definition:

- ✓ A **set** is a collection of well defined objects.
- ✓ If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that $a \in A, b \in B$ is called **the Cartesian Product of A and B**, and is denoted by $A \times B$. Thus $A \times B = \{(a, b) | a \in A, b \in B\}$

Note:

- ➢ $A \times B$ is the set of all possible ordered pairs between the elements of A and B such that the first coordinate is an element of A and the second coordinate is an element of B.
- ➢ $B \times A$ is the set of all possible ordered pairs between the element of A and B such that the first coordinate is an element of B and the second coordinate is an element of A
- ➢ If $a = b$, then $(a, b) = (b, a)$.
- ➢ The "Cartesian product" is also referred as "cross product"
- ➢ In general $A \times B \neq B \times A$, but $n(A \times B) = n(B \times A)$
- ➢ $A \times B = \emptyset$ if and only if $A = \emptyset$ or $B = \emptyset$
- ➢ If $n(A) = p$ and $n(B) = q$ then $n(A \times B) = pq$
- ➢ The set of all points in the Cartesian plane can be viewed as the set of all ordered pairs (x, y) where x, y are real numbers. In fact $\mathbb{R} \times \mathbb{R}$ is the set of all points which we call as the Cartesian plane.
- ➢ Distributive property of Cartesian product:
  (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
  (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- ➢ $A \times B$ represent a shape in two dimensions and $A \times B \times C$ represent an object in three dimensions.

Exercise 1.2

Concept corner

Definition: Let A and B be any two non-empty sets. A **relation (R)** from A to B is a subset of $A \times B$ satisfying some specified conditions. If $x \in A$ is related to $y \in B$ through R, then we write it as xRy.

xRy if and only if $(x, y) \in R$

- ✓ The domain of the relation $R = \{x \in A | xRy$, for some $y \in B\}$
- ✓ The co-domain of the relation R is B
- ✓ The range of the relation $R = \{y \in B | xRy$, for some $x \in A\}$

Note:

- ➢ A relation may be represented algebraically either by the roster method or by the set builder method.

Exercise 1.3

Concept corner

Definition: A relations f between two non-empty sets X and Y is called a **function** from X to Y if, for each $x \in X$ there exists only one $y \in Y$ such that $(x, y) \in f$.That is, $f = \{(x, y)/ \text{ for all } x \in X, \ y \in Y\}$

Note:

- ✓ If $f: X \rightarrow Y$ is a function then, the set X is called the domain, f and the set Y is called its co-domain.
- ✓ A function is also called as a mapping or transformation.
- ✓ $f: X \rightarrow Y$ is a function only if
  i) every element in the domain of f has an image.
  ii) the image is unique.
- ✓ If A and B are finite sets such that $n(A) = p$, $n(B) = q$ then the total number of functions that exist between A and B is q^p
- ✓ If $f(a) = b$, then b is called **image** of a under f and a is called a **pre-image** of b.
- ✓ The set of all images of the elements X under f is called the **range** of f.
- ✓ Describing domain of a function
  (i) Let $(x) = \frac{1}{1+x}$. If $x = -1$ then $f(-1)$ is not defined. Hence f is defined for all real numbers except at $x = -1$. So, domain of f is $\mathbb{R} - \{-1\}$
  (ii) Let $(x) = \frac{1}{x^2-5x+6}$, if $x = 2,3$ then $f(2)$ and $f(3)$ are not defined. Hence f is defined for all real numbers except at $x = 2$ and 3. So domain of $f = \mathbb{R} - \{2,3\}$

- ➢ An arrow diagram is a visual representation of a relation.
- ➢ If $n(A) = p, n(B) = q$ then the total number of relations that exist from A to B is 2^{pq}.
- ➢ A relation which contains no elements is called a "**Null relation**"

Exercise 1.4

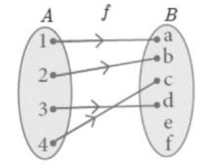
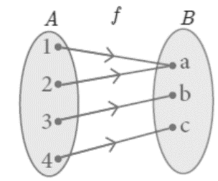
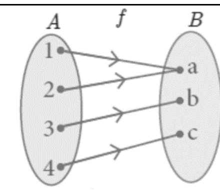
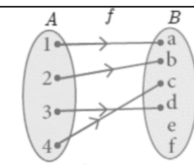
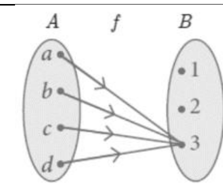
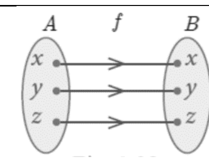
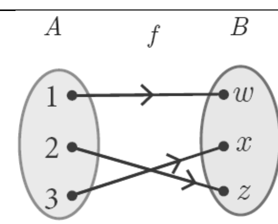
Concept corner

Note: Any equation represented in a graph is usually called a curve.

- ✓ **Representation of functions**
  a) a set of ordered pairs   b) a table form
  c) An arrow diagram   d) a graphical form.
- ✓ **Vertical line test:** A curve drawn in a graph represents a function, if every vertical line intersects the curve in at most one point.
- ✓ **Horizontal Line Test:** A function represented in a graph in one – one, if every horizontal line intersects the curve in at most one point.
- ✓ Every function can be represented by a curve in a graph. But not every curve drawn in a graph will represent a function.
- ✓ If $f: A - B$ is an onto function then, the range of $f = B$

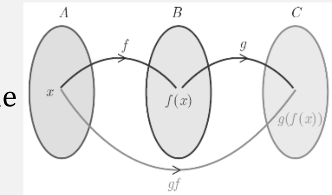
Note: A one-one and onto function is also called a one-one correspondence.

Types of functions

Sl.No	Name	Definition	Mapping Example
1	One-One function (Injection)	A function $f: A \rightarrow B$ is called one-one function if distinct elements of A have distinct images in B.	
2	Many-one function	A function $f: A \rightarrow B$ is called many-one function if two or more elements of A have same image in B	
3	Onto function (Surjection)	A function $f: A \rightarrow B$ is said to be onto function if the range of f is equal to the co-domain of f.	
4	Into function	A function $f: A \rightarrow B$ is called an into function if there exists at least one element in B which is not the image of any element of A	
5	Constant function	A function $f: A \rightarrow B$ is called a constant function if the range of f contains only one element. That is, $f(x) = c$ for all $x \in A$ and for some fixed $c \in B$.	
6	Identity function	Let A be a non-empty set. Then the function $f: A \rightarrow A$ defined by $f(x) = x$ for all $x \in A$ is called an identity function on A and is denoted by I_A.	
7	Bijection	If a function $f: A \rightarrow B$ is both one-one and onto, then f is called a bijection from A to B	
8	Real – Valued function	A function $f: A \rightarrow B$ is called a real valued function if the range of f is a subset of the set of all real numbers R. That is $f(A) \subseteq R$	

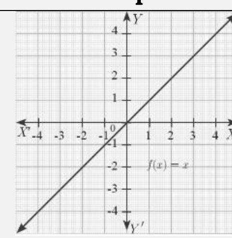
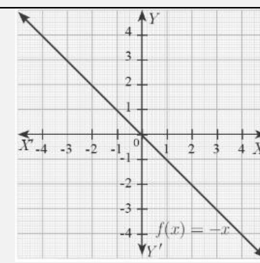
Exercise 1.5

Concept corner

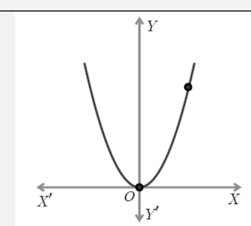
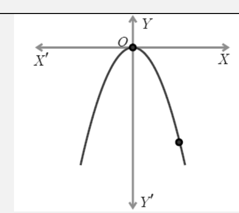


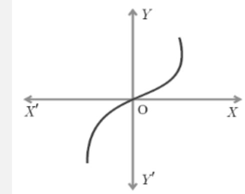
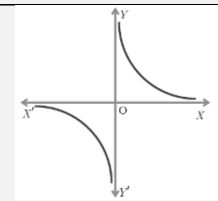
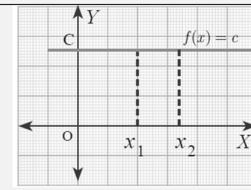
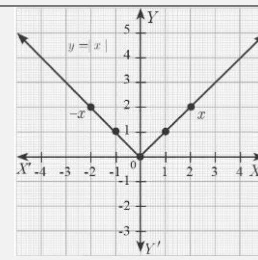
Definition: Let $f: A \to B$ and $g: B \to C$ be two functions. Then the composition of f and g denoted by $g \circ f$ is defined as the function $g \circ f(x) = g(f(x))$ for all $x \in A$.

- ✓ The composition $g \circ f(x)$ exists only when range of f is a subset of g
- ✓ $f \circ g \neq g \circ f$ Composition of function is not commutative.
- ✓ Composition of three functions is always associative. That is $f \circ (g \circ h) = (f \circ g) \circ h$.
- ✓ A function $f: R \to R$ defined by $f(x) = mx + c,\ \ m \neq 0$ is called a **linear function.** Some specific linear functions and their graphs are given below.

No.	Function	Domain and Definition	Graph
1	The identity function	$f: R \to R$ defined by $f(x) = x$	
2	Additive inverse function	$f: R \to R$ defined by $f(x) = -x$	

- ✓ A function $f: R \to R$ defined by $f(x) = ax^2 + bx + c\ \ (a \neq 0)$ is called a **quadratic function.**

Function, Domain, Range and Definition	Graph
$f: R \to R$ defined by $f(x) = x^2, x \in R, f(x) \in [0, \infty\ )$	
$f: R \to R$ defined by $f(x) = -x^2, x \in R.f(x) \in (-\infty, 0]$	

A function $f: R \rightarrow R$ defined by $f(x) = ax^3 + bx^2 + cx + d$ $(a \neq 0)$ is called a **cubic function.**	
A function $f: R - \{0\} \rightarrow R$ defined by $f(x) = \frac{1}{x}$ is called a **reciprocal function.**	
A function $f: R \rightarrow R$ defined by $f(x) = c$ for all $x \in R$ is called a **constant function.**	
Modulus or Absolute Valued Function: $f: R \rightarrow [0, \infty)$ defined by $f(x) = \lvert x \rvert = \begin{cases} x\ ; x \geq 0 \\ -x\ ; x < 0 \end{cases}$	

✓ Modulus function is not a linear function but it is composed of two linear functions x and $-x$